A Crash Course in Good and Bad Controls

Carlos Cinelli (UW), Andrew Forney (LMU), and Judea Pearl (UCLA)

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The study controlled for socioeconomic factors like a participant's income, education level and mobility, said Andrew Steptoe, a co-author of the study and the head of University College London's research department of behavioral science and health.

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Some variables are bad controls and should not be included in a regression model, even when their inclusion might be expected to change the short regression coefficients. Bad controls are variables that are themselves outcome variables in the notional experiment at hand. That is, bad controls might just as well be dependent variables too. Good controls are variables that we can think of having been fixed at the time the regressor of interest was determined.

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Our goal today is to learn the main lessons of these general graphical criteria *through simple examples*.

We will see how causal diagrams can make otherwise difficult problems very easy to solve, *by* – *literally* – *simple inspection of a diagram*.

Birth-weight paradox: Infants born to smokers were found to have higher risks of mortality than infants born to non-smokers. However, among infants with low birth-weight (LBW), this relationship was <u>reversed</u>. This reversal of effects has created many controversies in epidemiology—does it mean that maternal smoking is beneficial for LBW infants?

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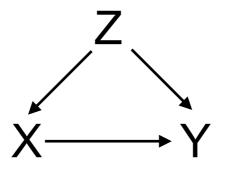
These are all related to "bad controls."

Preliminaries – Causal Diagrams

Causal diagrams have become popular in the social and health sciences for explaining and resolving difficult problems in causal inference in a rigorous, yet accessible manner.

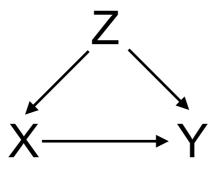
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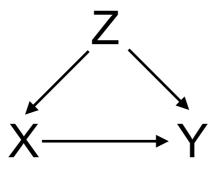
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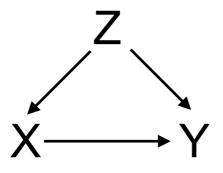


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2. <u>Arrows</u> denote a (possible) <u>direct causal effect</u> between one variable on another. For instance, the arrows $X \rightarrow Y$ and $Z \rightarrow Y$ state that both the drug and income could possibly affect health.

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Note that <u>no parametric assumptions</u> need to be made regarding the functional form of the causal relationships, nor the distribution of variables.

Building blocks of a causal diagram

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Let us understand better each of these forms of association, and when they are <u>closed</u> or <u>opened</u>.

1. <u>Mediators:</u> X *causally* affects Y through Z. This is a <u>causal</u> path from X to Y. If left untouched, the path is open.

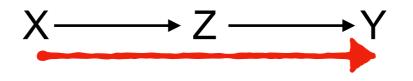
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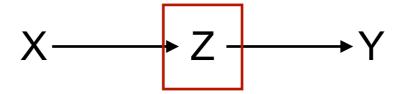
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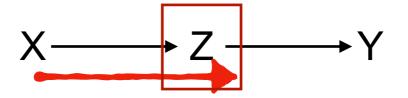
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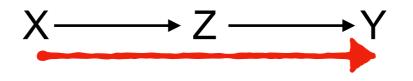
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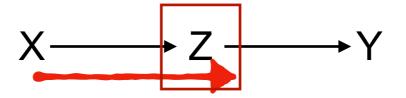
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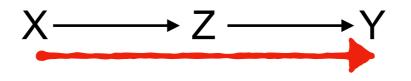


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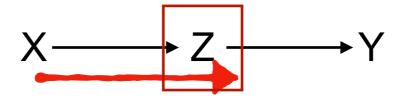


Example: Consider a drug (X) that affects a health outcome (Y) by lowering blood pressure (Z).

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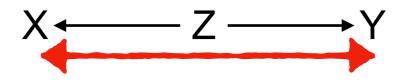
Conditioning on blood pressure <u>blocks</u> the mechanism through which the drug affects health. Thus you will not see any association between drug use and health status among those with the same level of blood pressure.

2. <u>**Common causes:**</u> X and Y share a common cause Z (aka confounder, or "back-door" path). Left on its own, it is open, and it induces a <u>non-causal</u> (spurious) association between X and Y.

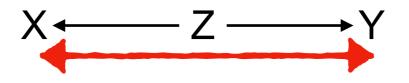
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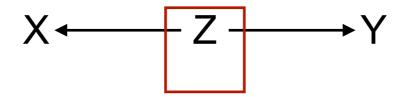
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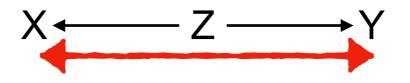
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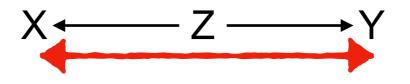
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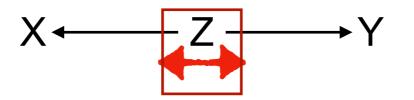
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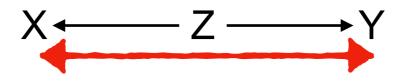


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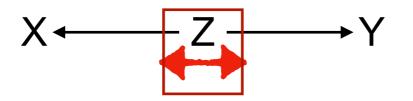


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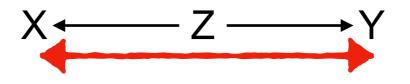
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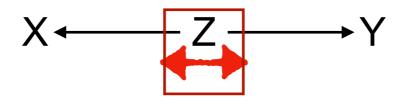
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Conditioning on income (Z) <u>blocks</u> this spurious association.

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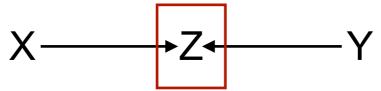
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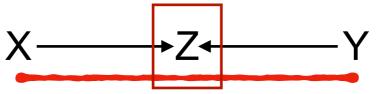
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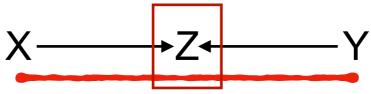
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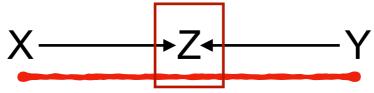


Example: Imagine beauty (X) and talent (Y) are independent in the general population. However, suppose that movie agencies only hire (Z=1) actors whose beauty + talent exceed a certain threshold.

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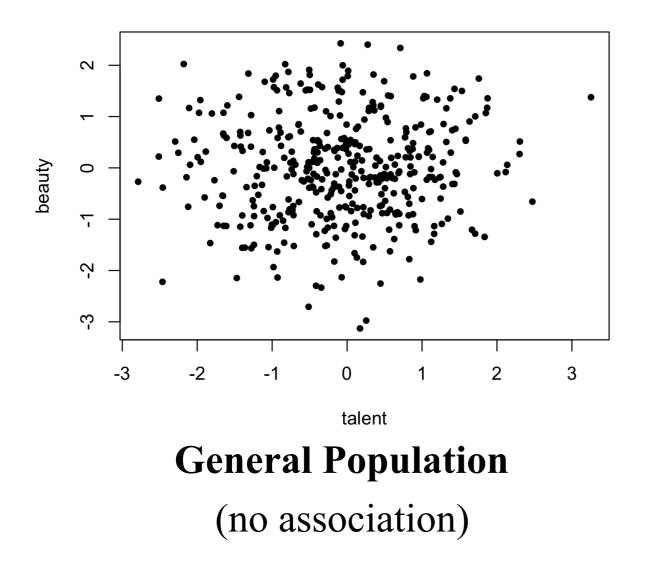
Even though there's no causal relationship between beauty and talent in the general population, you will see a *negative* association between beauty and talent *both among hired actors* (Z=1) and not hired actors (Z=0).



beauty <- rnorm(n)
talent <- rnorm(n)
hire <- (beauty + talent > 0)

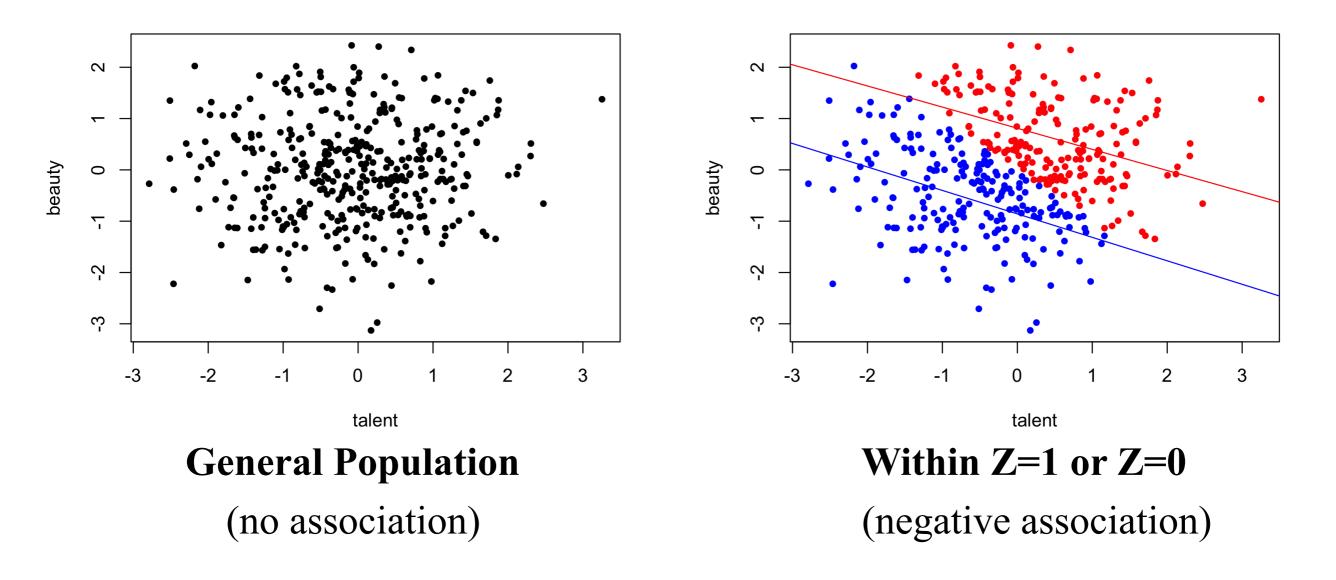


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Path	Path		Not conditioning on Z	Conditioning on Z
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Common Causes (Confounders)	X ← Z → Y	<u>Non-Causal</u> (Spurious)	Open	Closed
Common Effects (Colliders)	X →→ Z ∢ → Y	<u>Non-Causal</u> (Spurious)	Closed	Open

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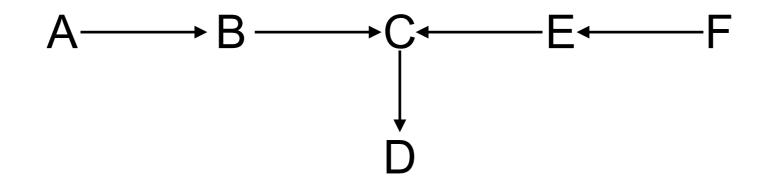
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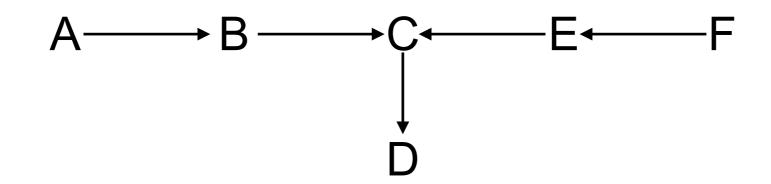
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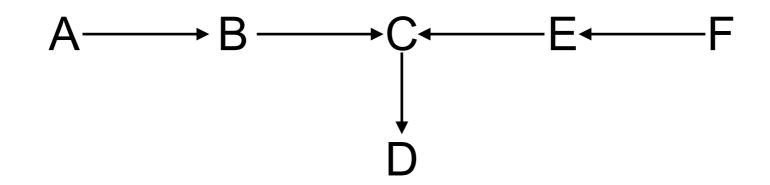
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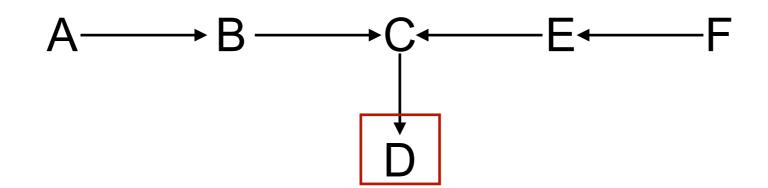


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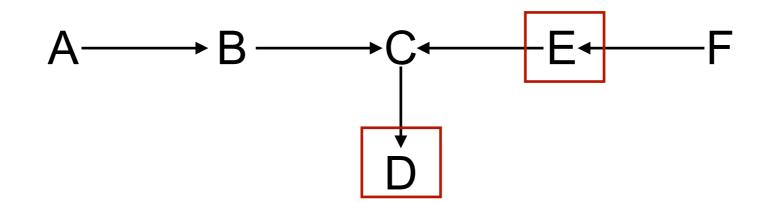


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Although simple, mastering this does require some practice. So let's apply those principles in very simple examples.

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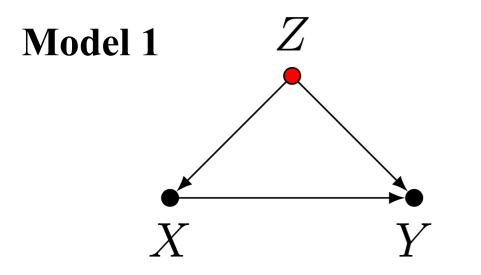
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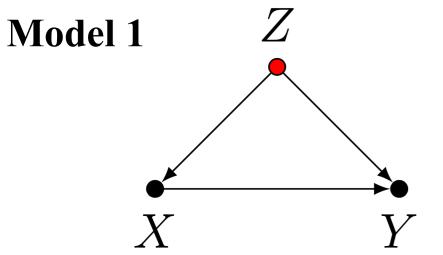
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Here will focus on *practicing our graphical skills*. Later we will see how these very simple models can help you make sense of real world scenarios.

"Good" Controls – Blocking backdoor paths

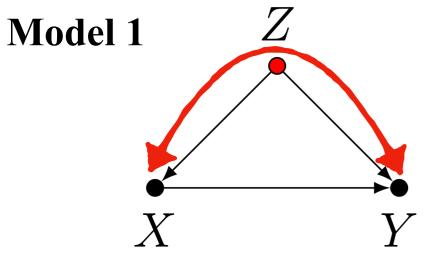


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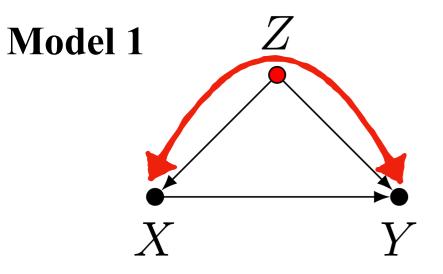


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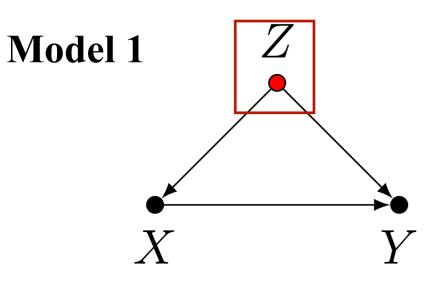
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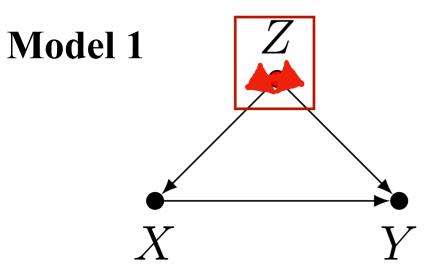
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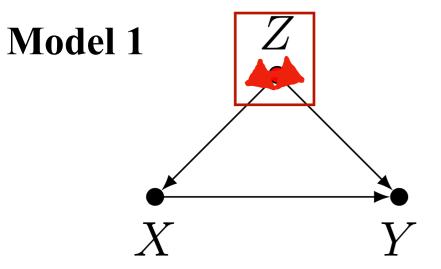
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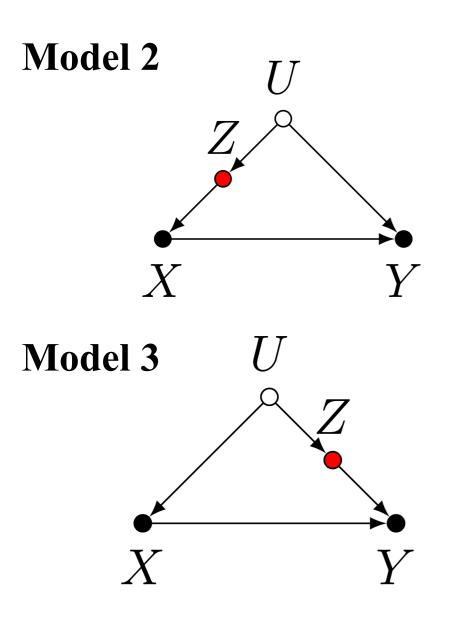
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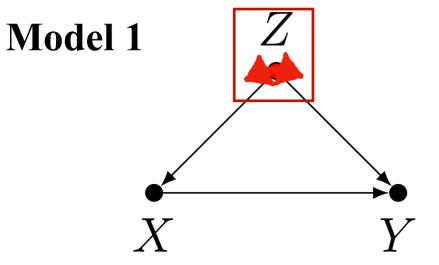


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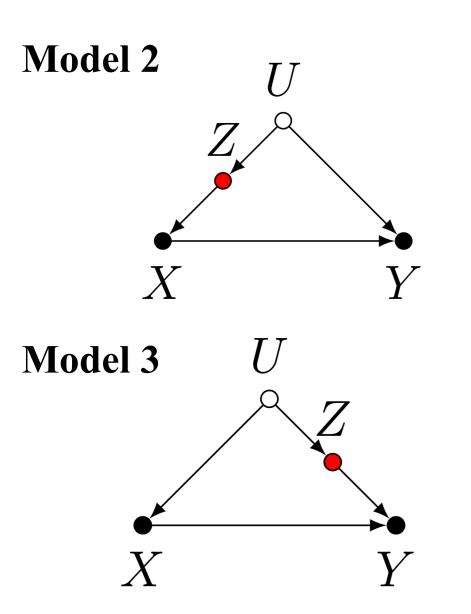
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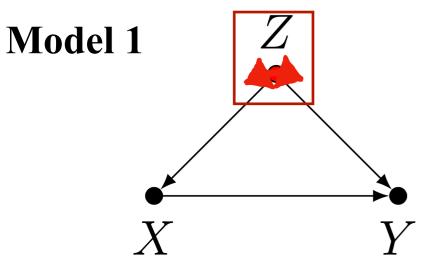


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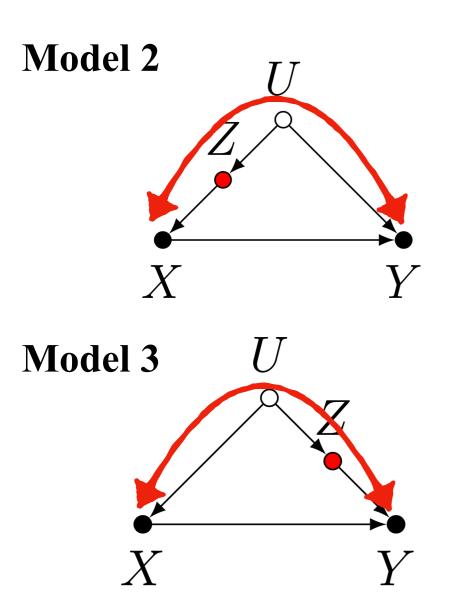


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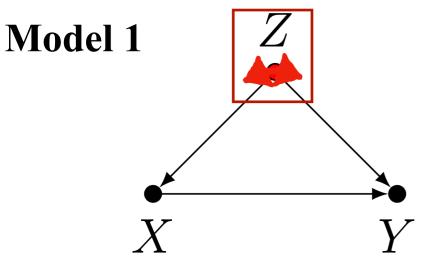


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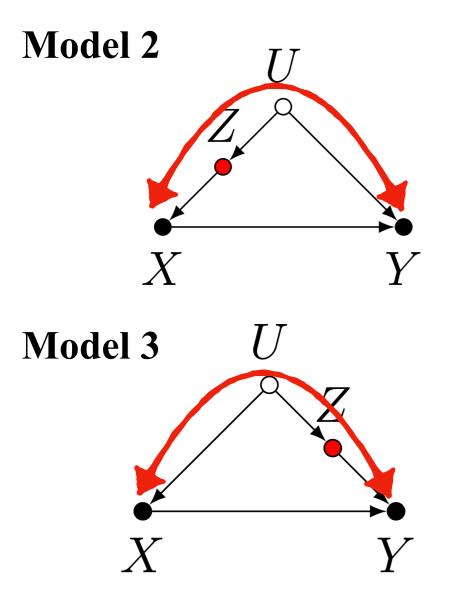


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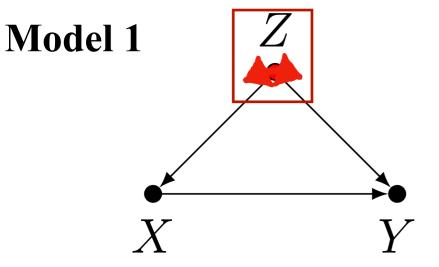
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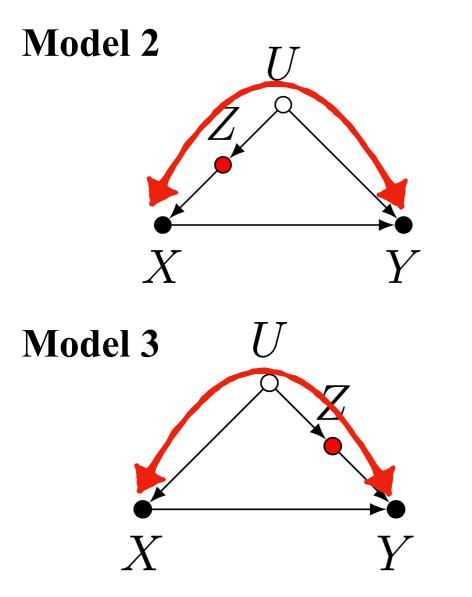
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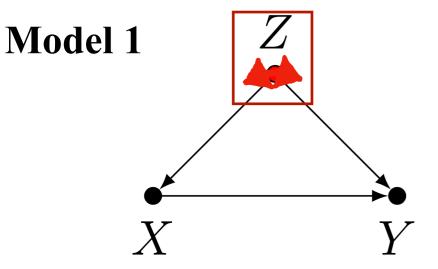
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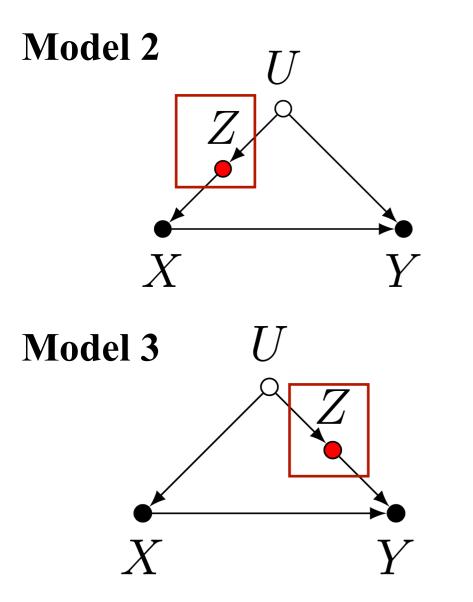
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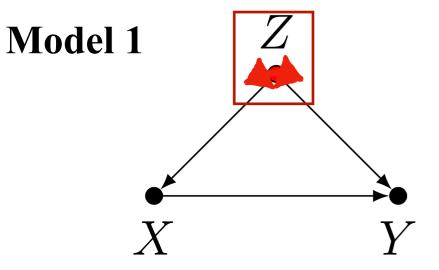
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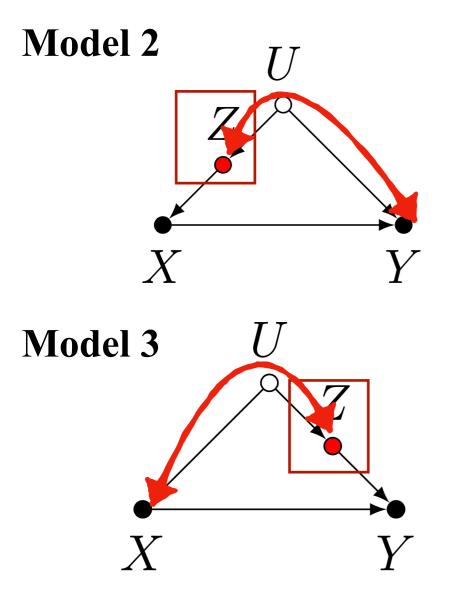
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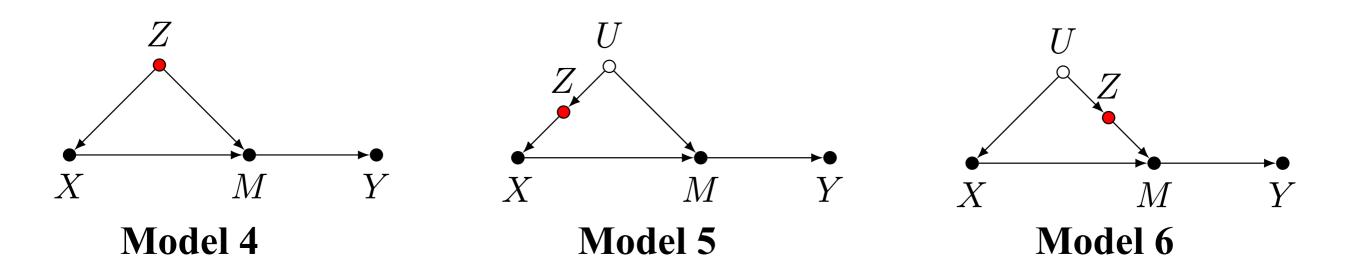
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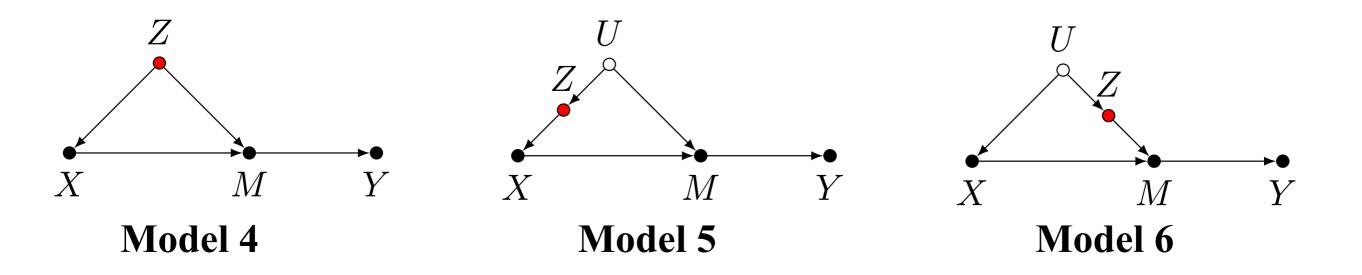
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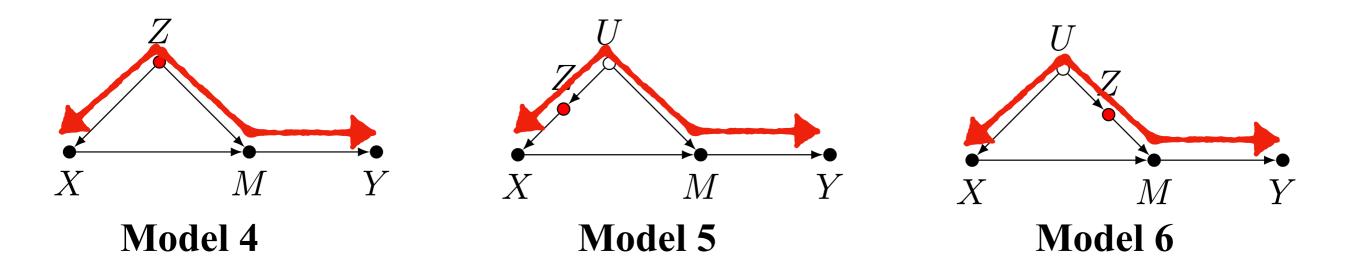
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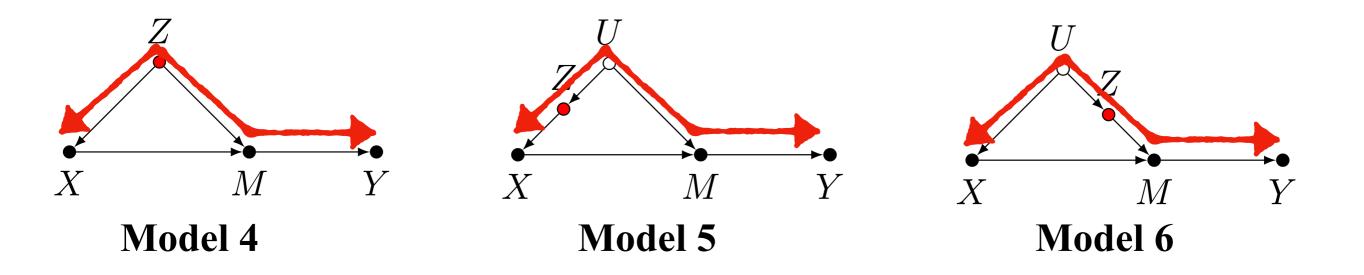
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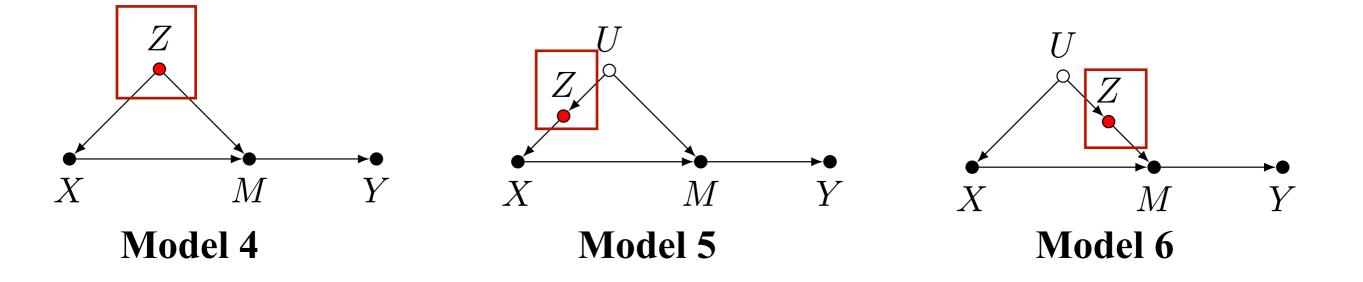


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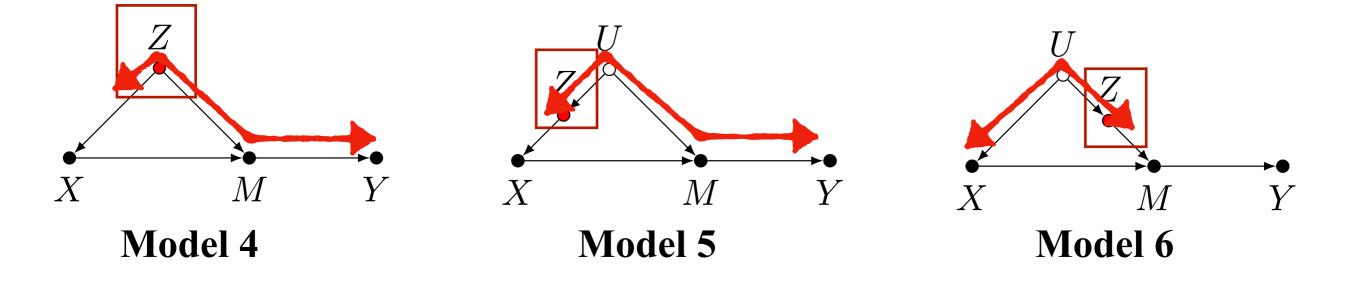


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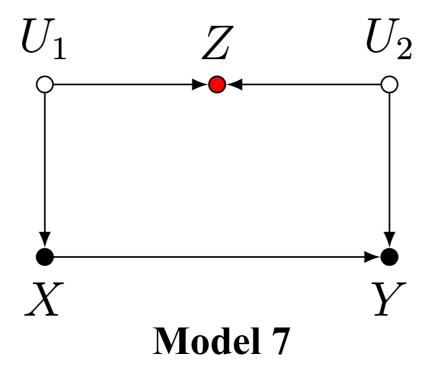
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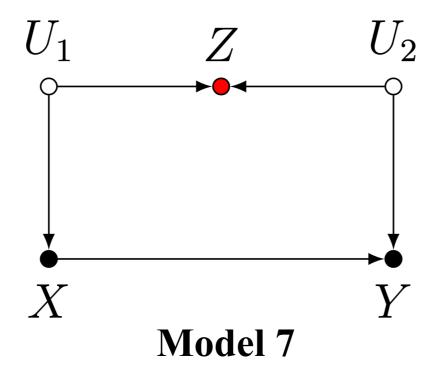
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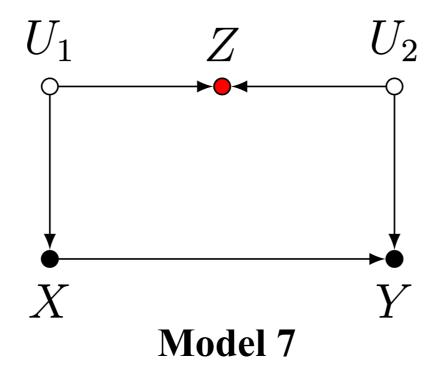


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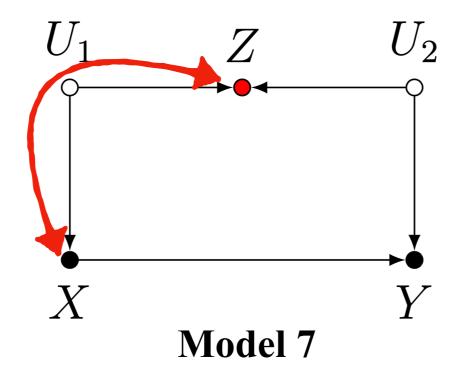
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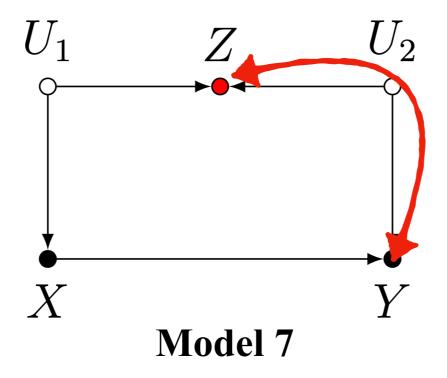
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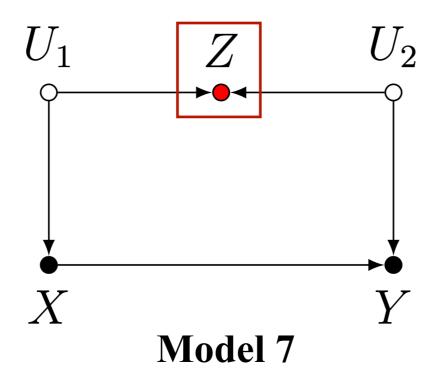
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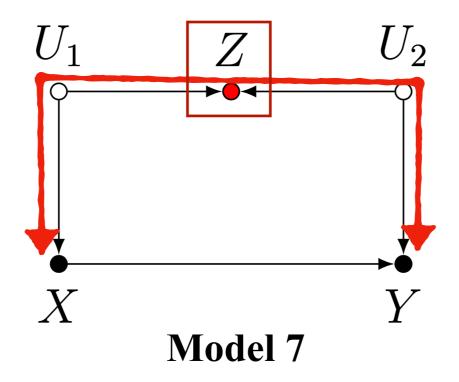


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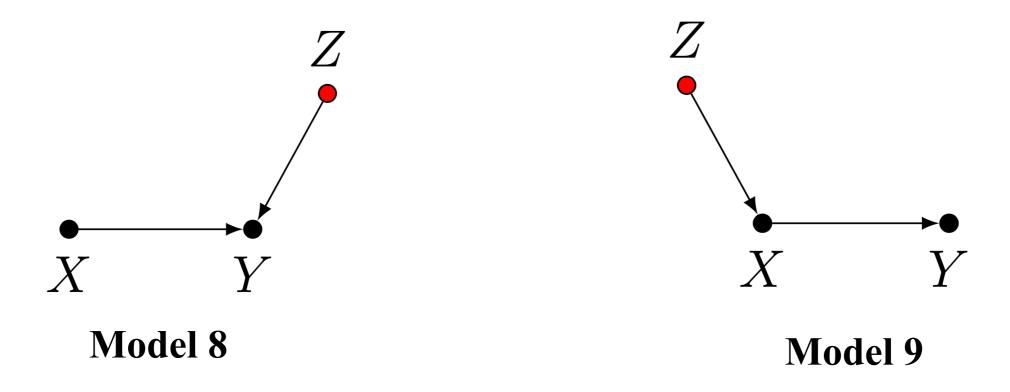
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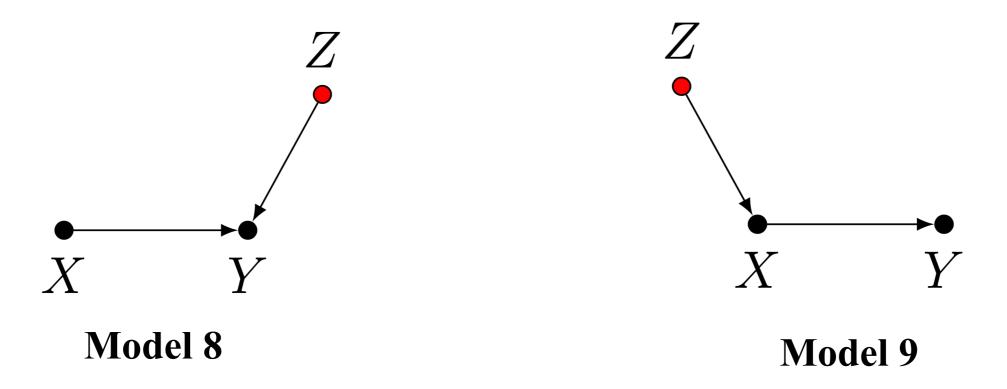


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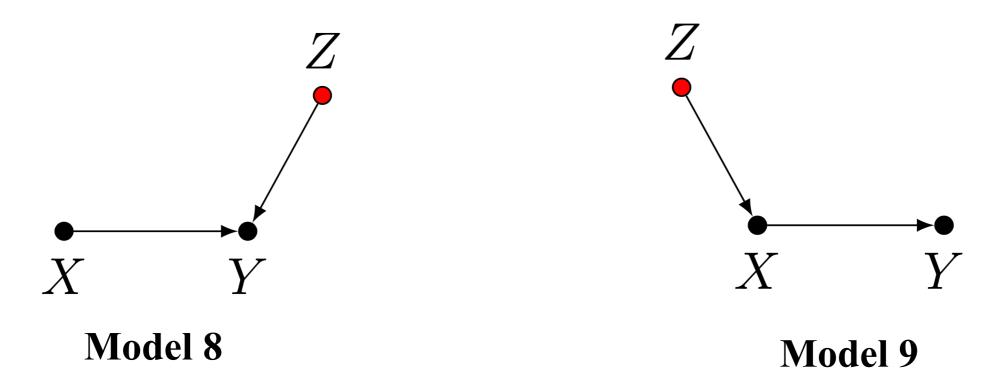
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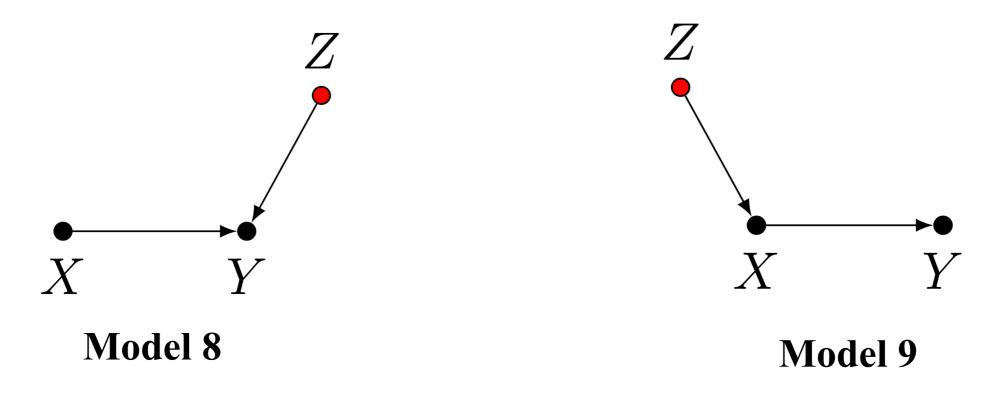


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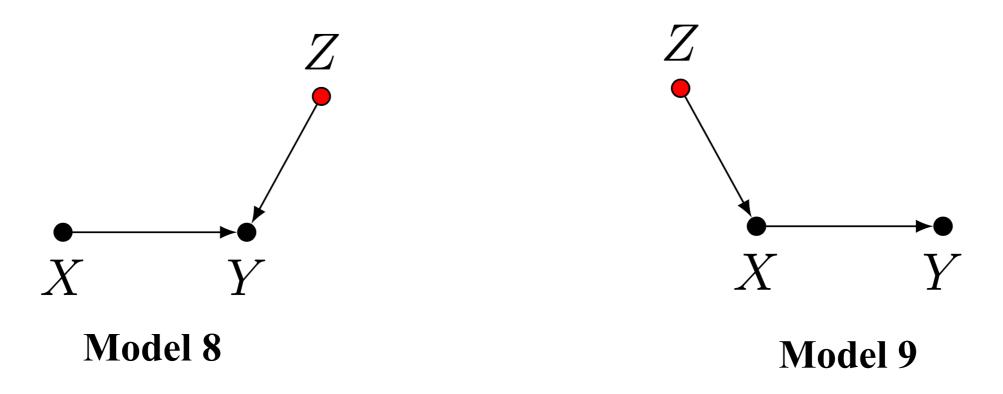
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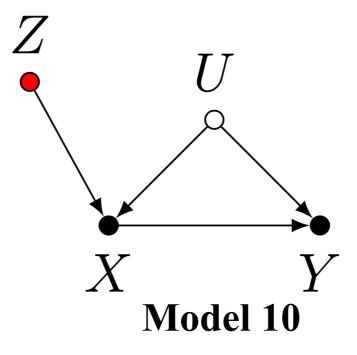


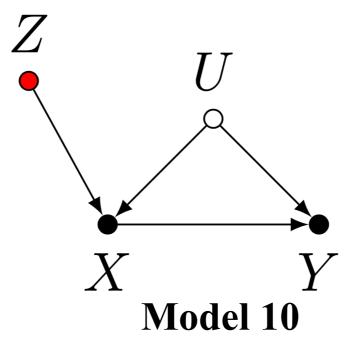
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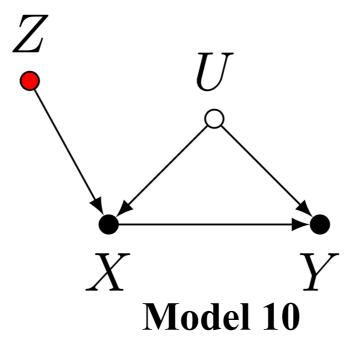
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Thus, in Model 8, Z improves the precision of the ATE estimate; Whereas in Model 9 Z hurts the precision of the ATE estimate.



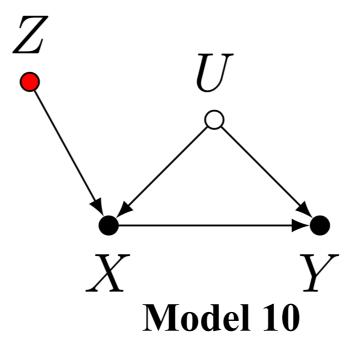


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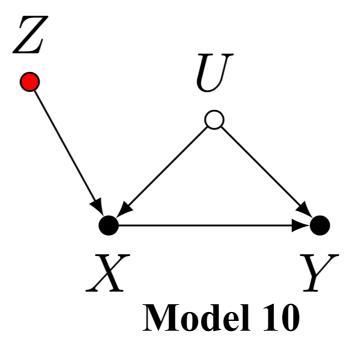
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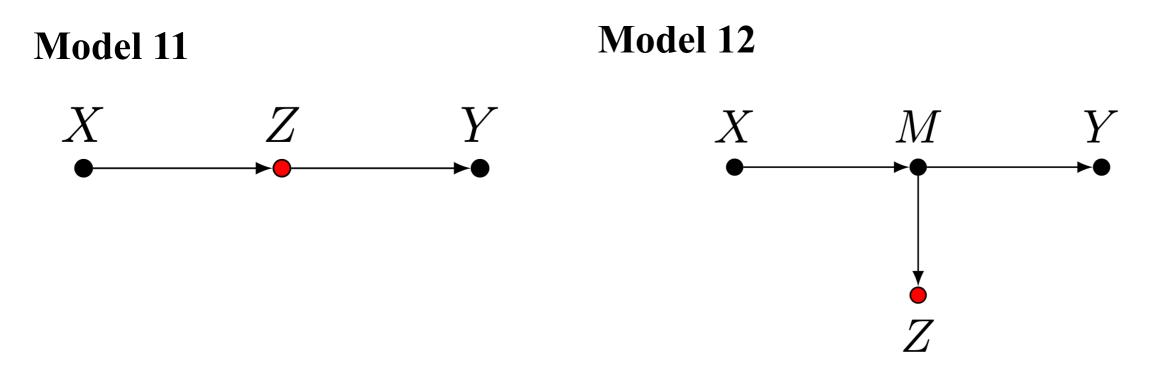
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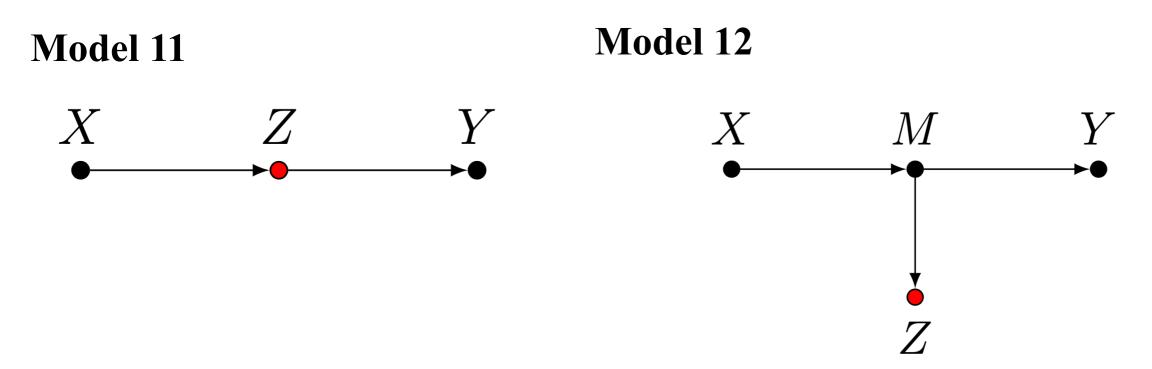
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However, analysis shows that adjusting for Z will not only fail to deconfound the effect of X on Y, but, in linear models, it will *amplify* any existing bias.

"Bad" Controls (overcontrol bias)

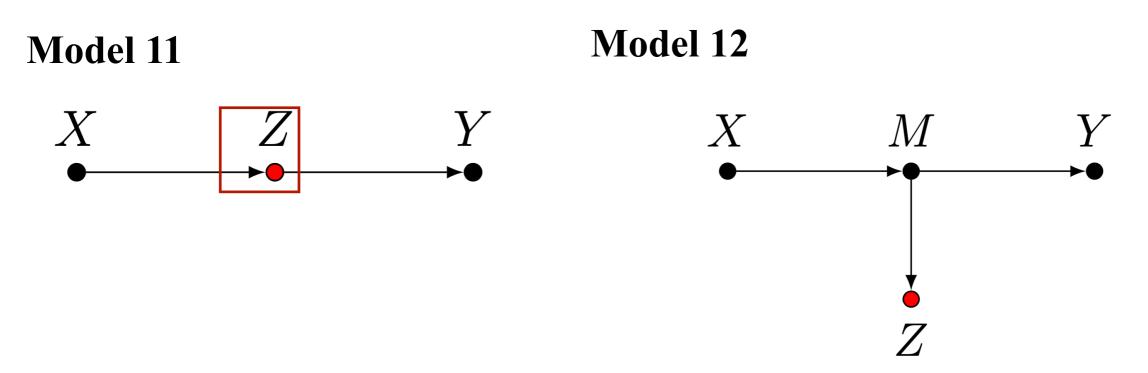


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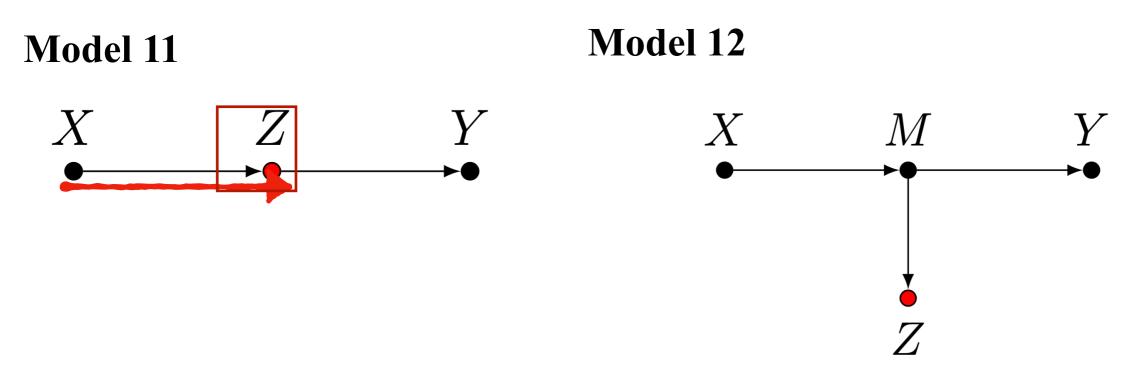
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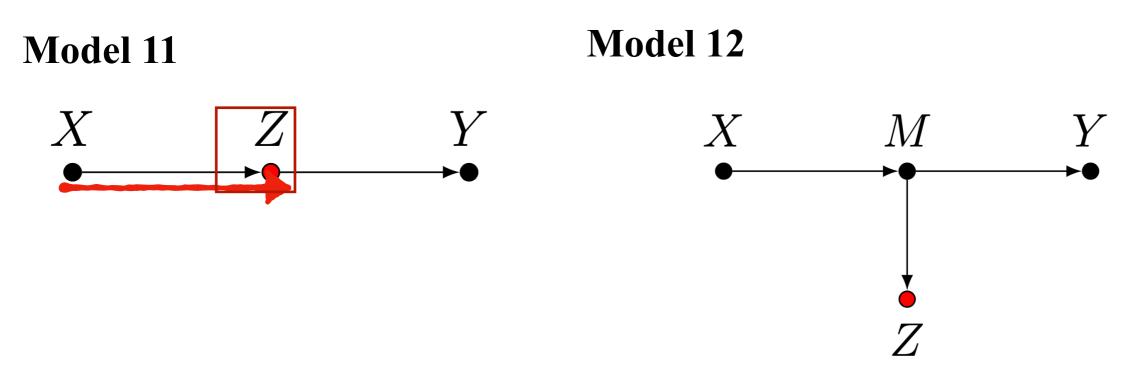
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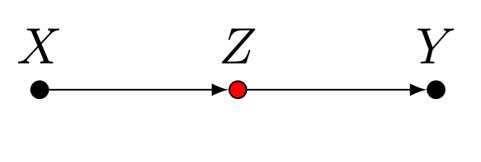
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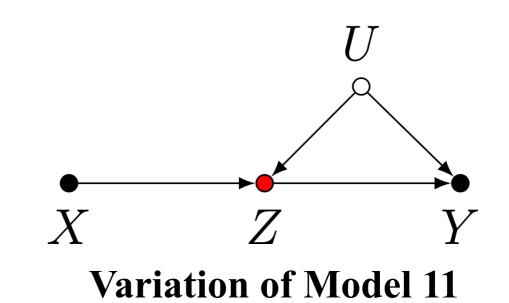
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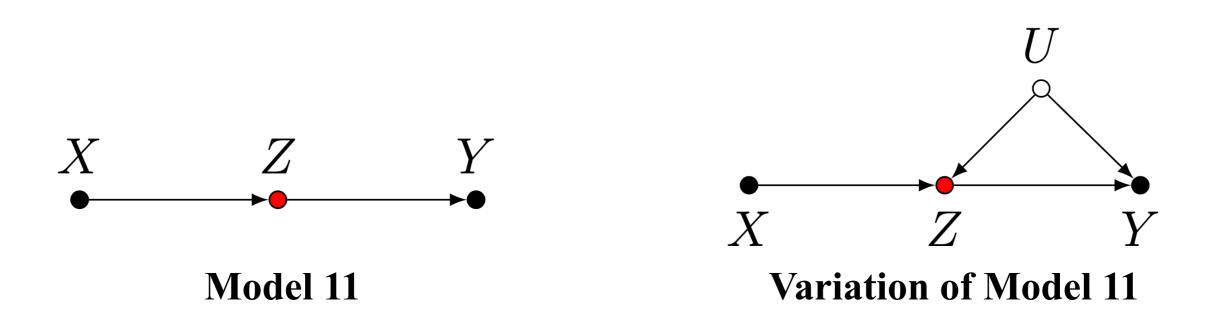
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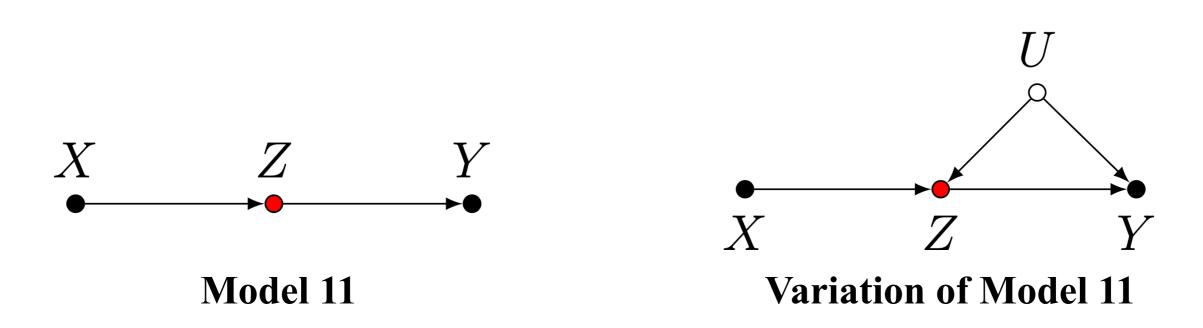


Model 11



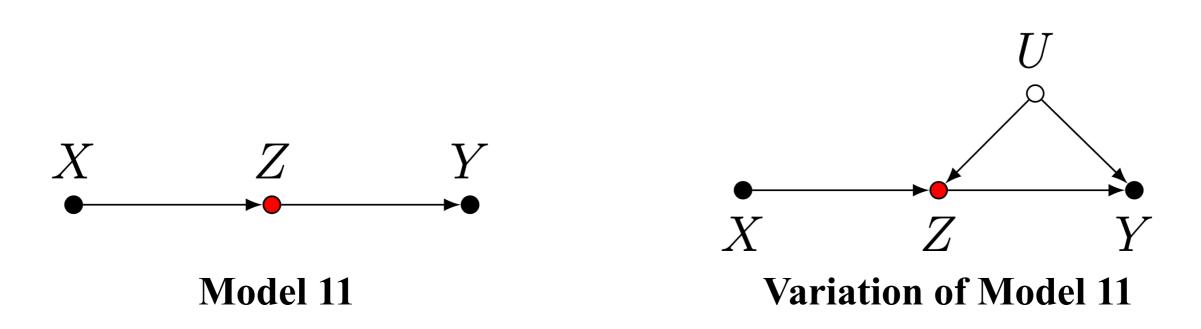


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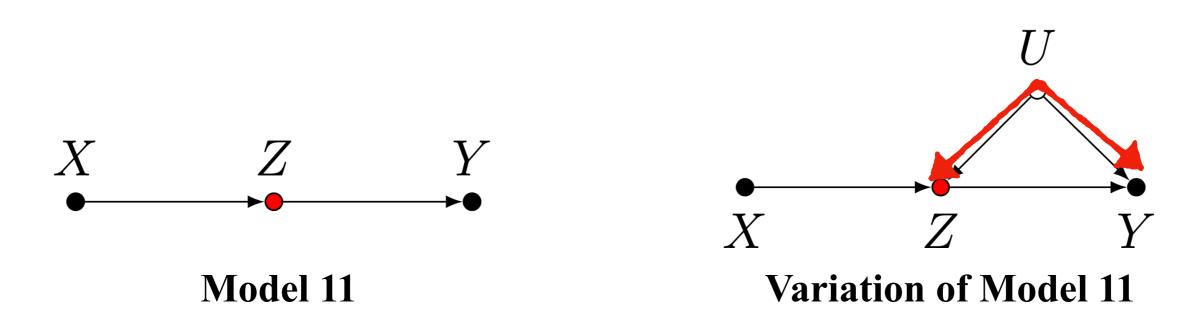
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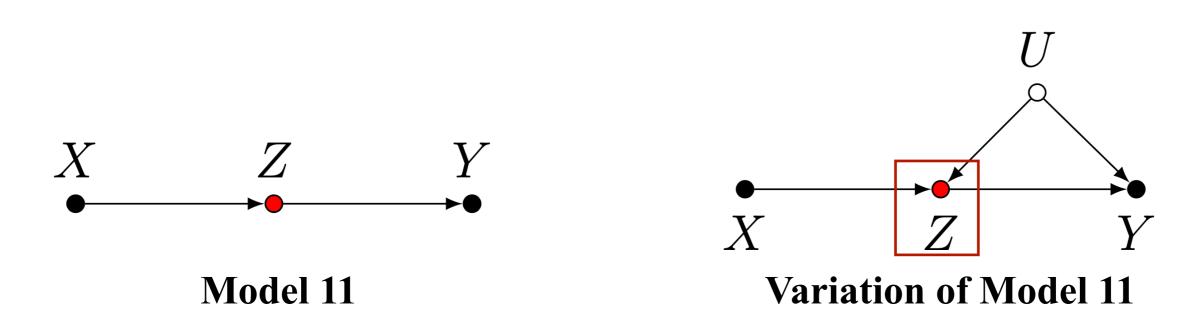
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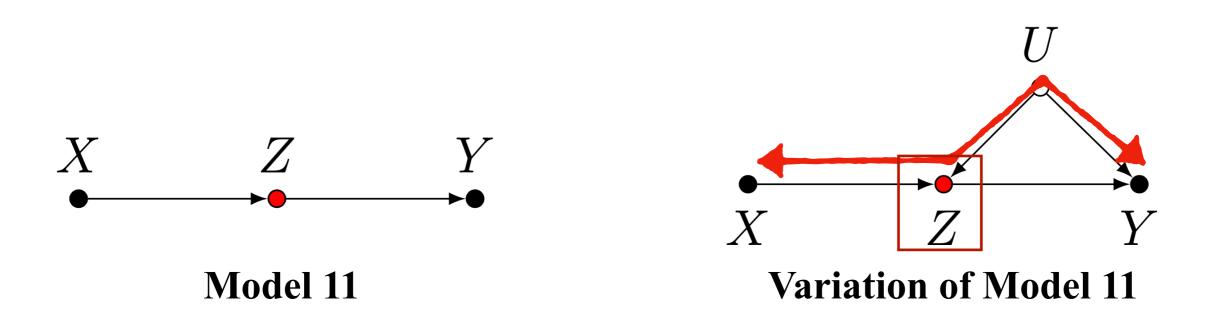


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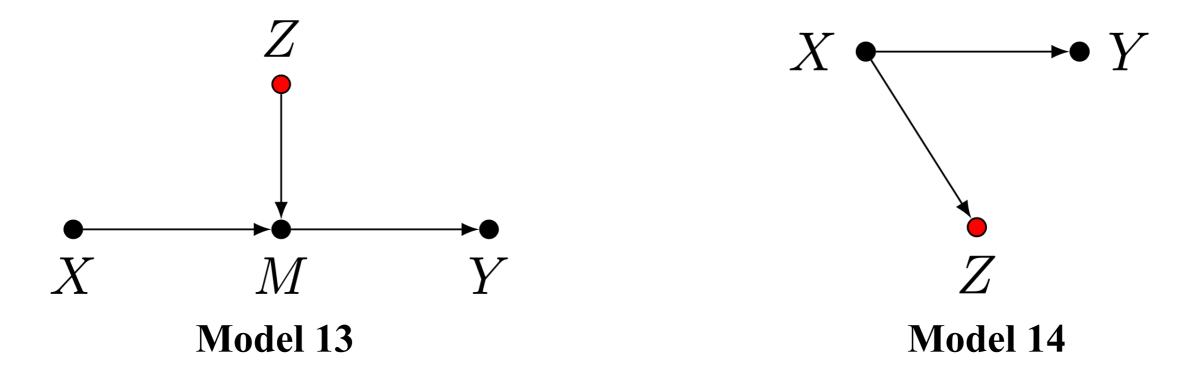


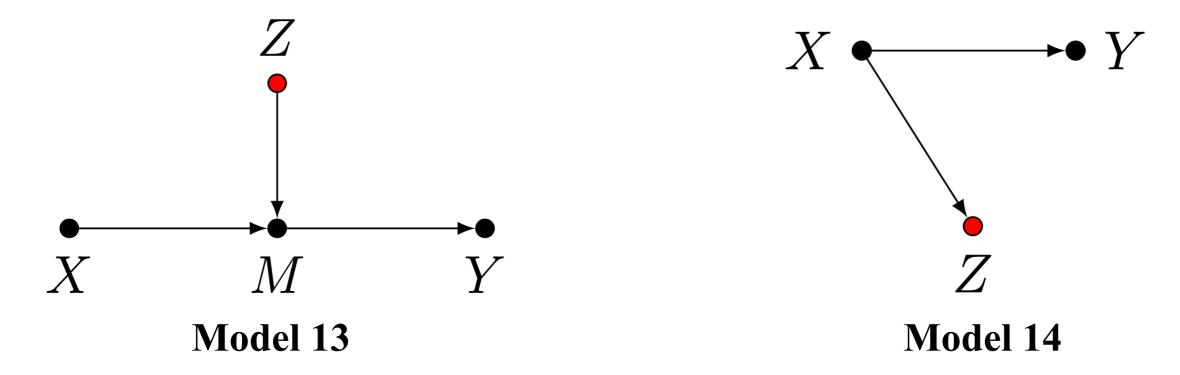
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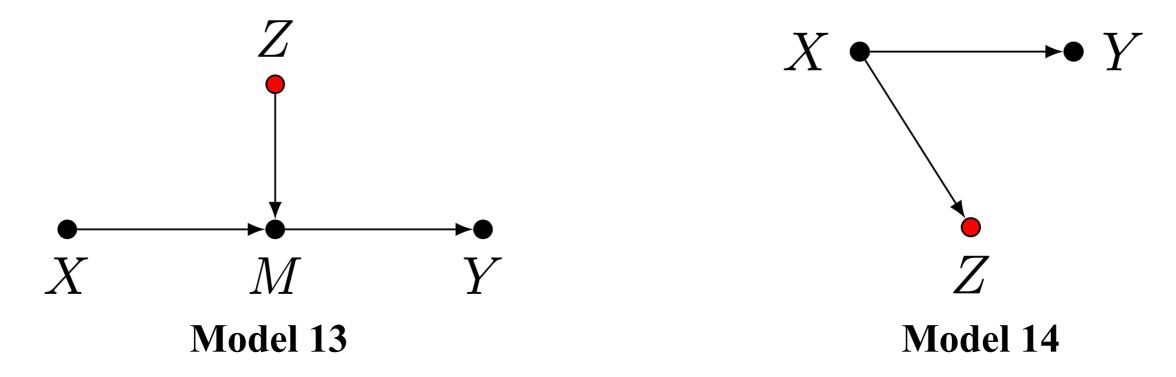
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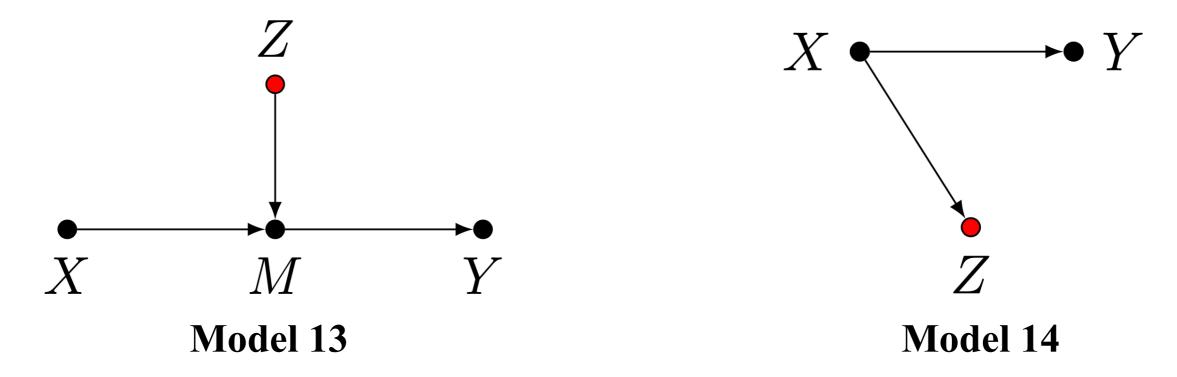


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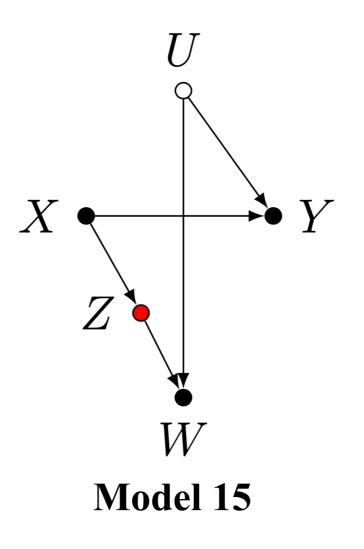
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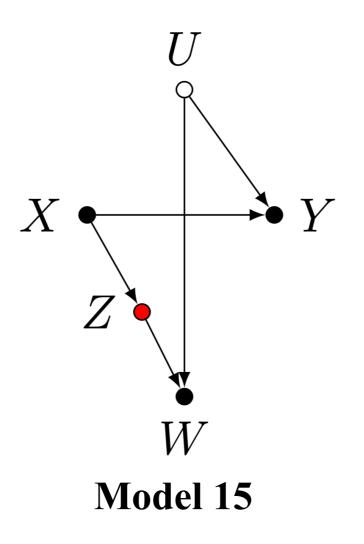


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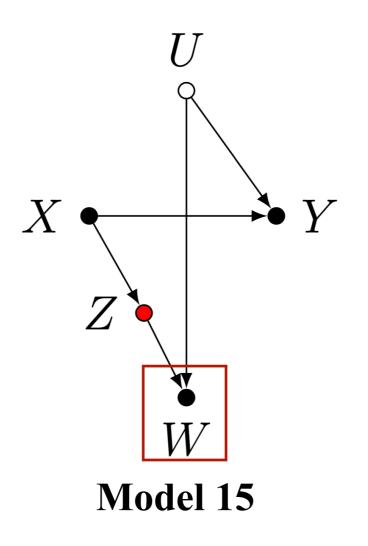
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Contrary to folklore, not all "post-treatment" variables are inherently bad controls. In Model 14, Z is post-treatment, and controlling for Z does not open any confounding paths between X and Y. However, as before, controlling for Z may *hurt* the precision of the ACE estimate in finite samples.



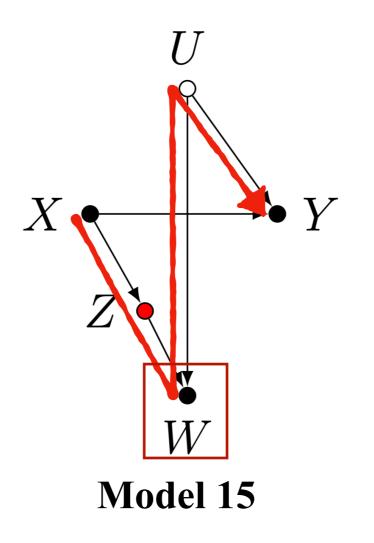


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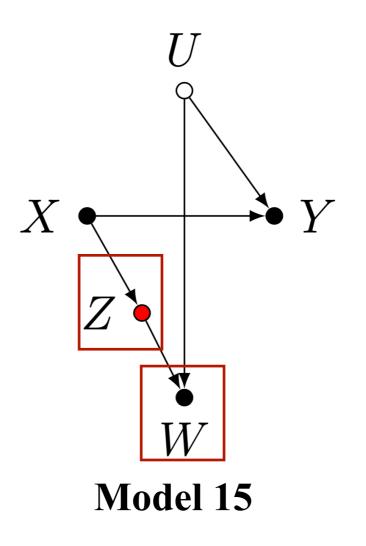
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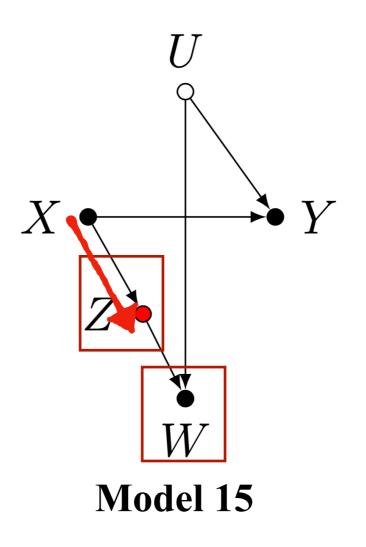
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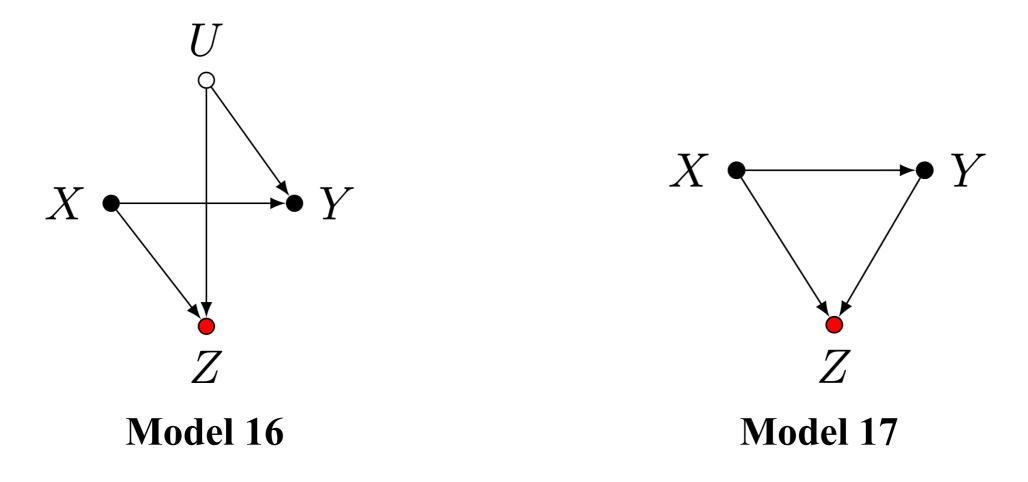
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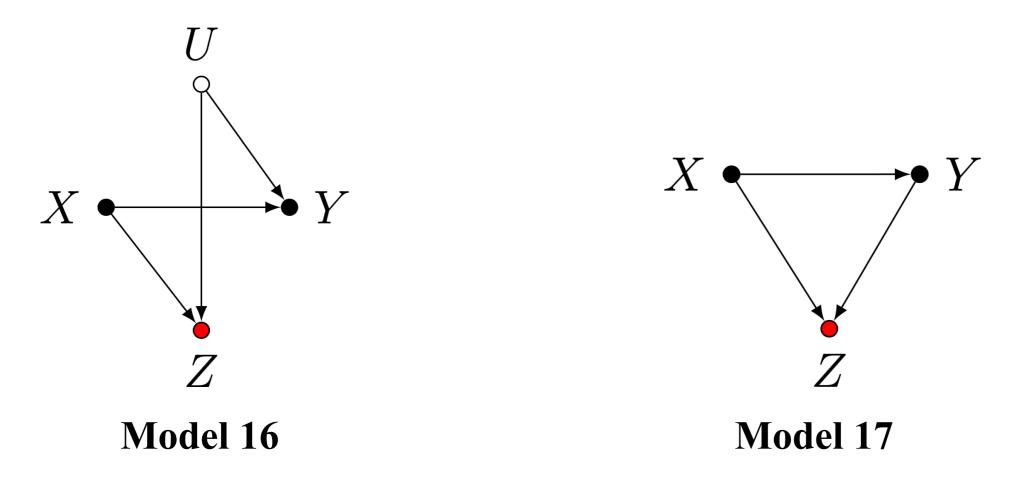


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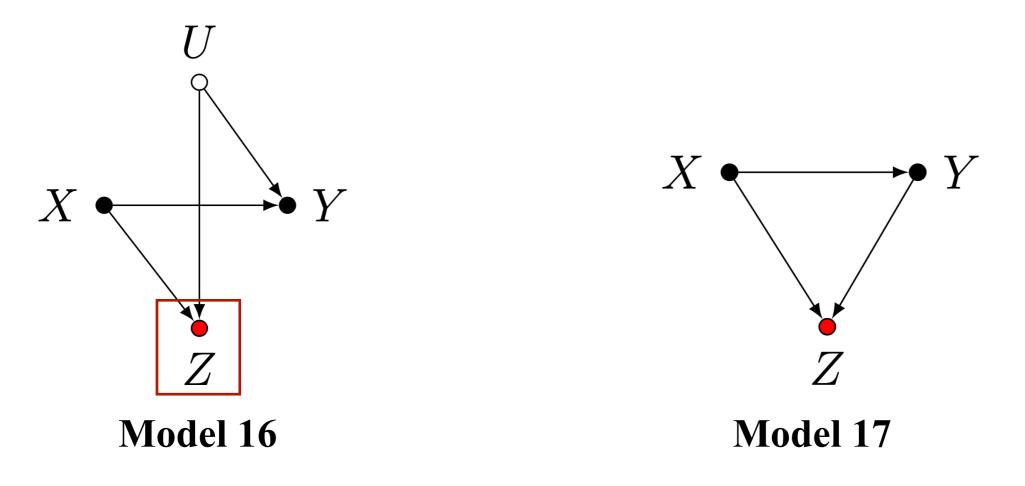
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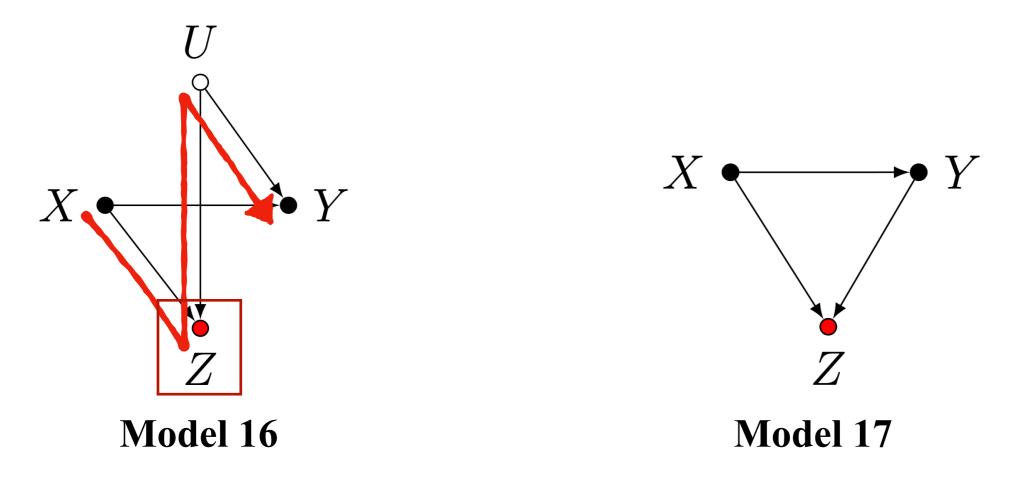


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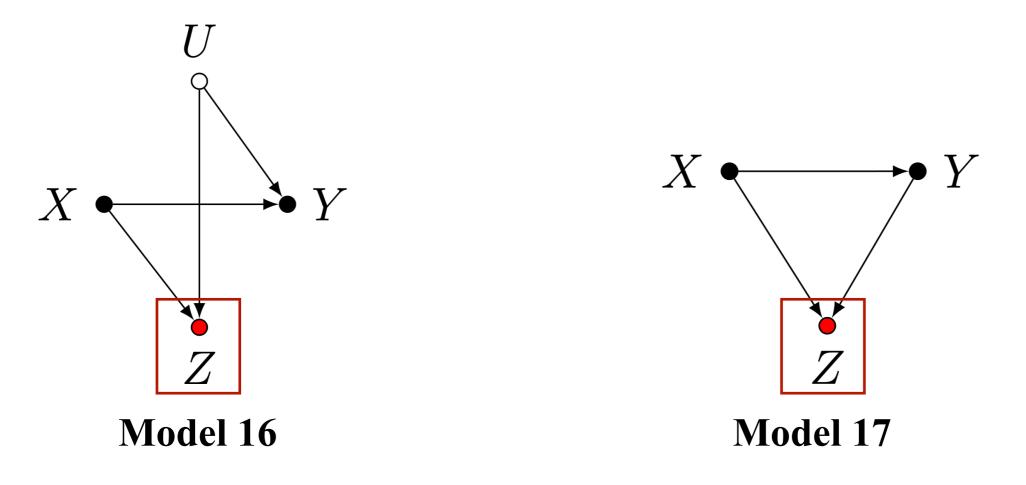
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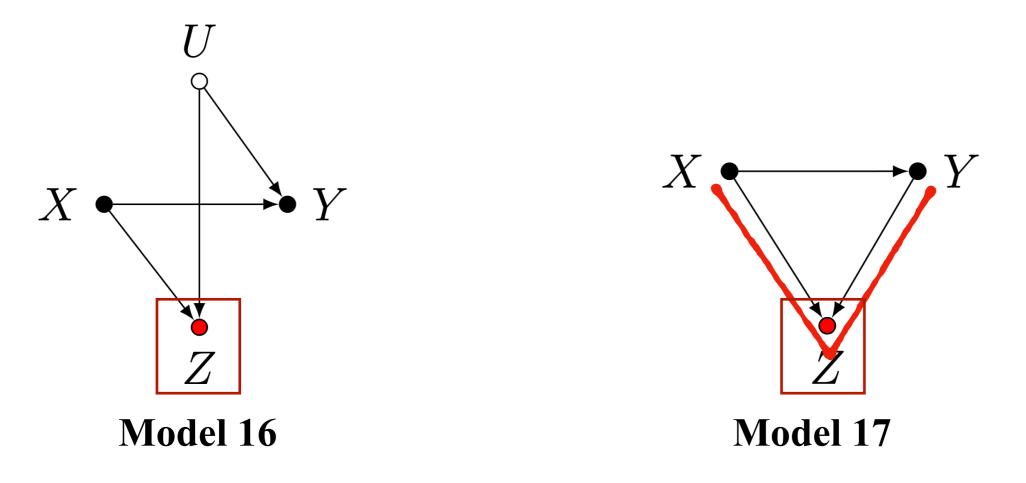
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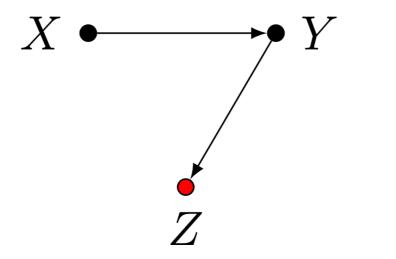
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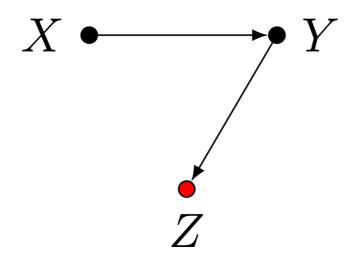


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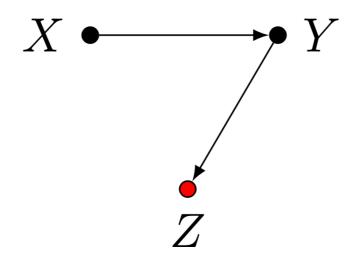
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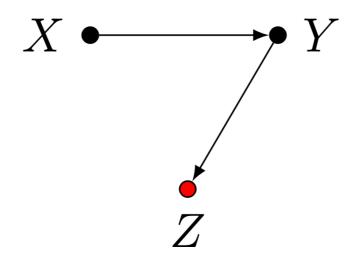
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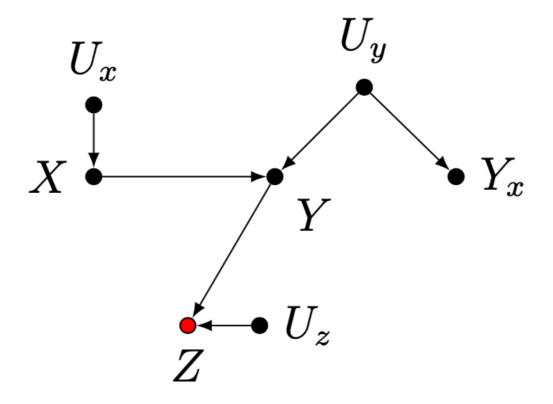


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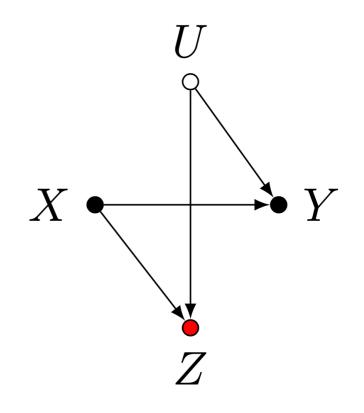
Some Real Examples of Bad Controls

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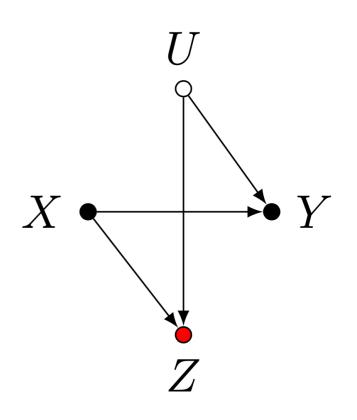
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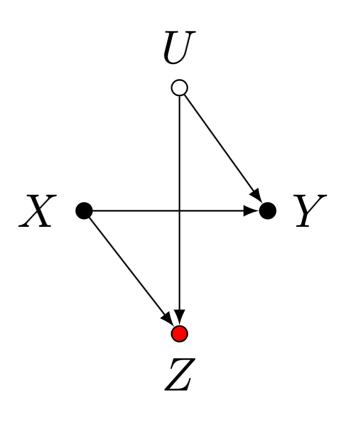
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- LBW infants of non-smokers need to have alternative causes for their LBW (such as malnutrition), and such causes could also lead to higher mortality.

U

Z

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Does better nutrition reduce the heights of adult men?

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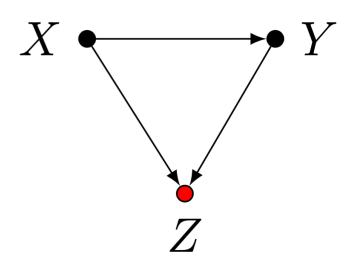
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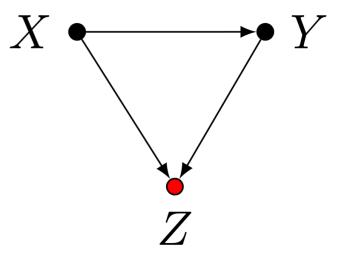


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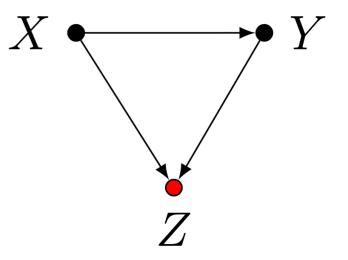
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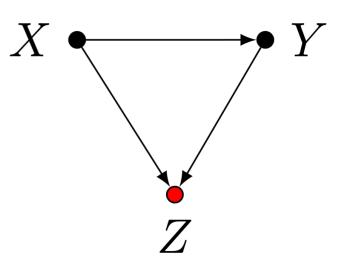
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Here one could argue that both childhood nutrition and adult height have pathways to committing a crime through socioeconomic opportunities, leading to selection bias.



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Jeffrey Wooldridge @jmwooldridge

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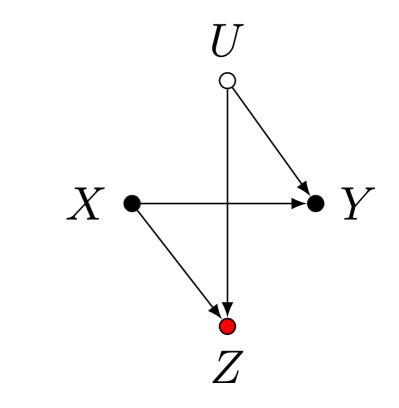


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We can illustrate this with Model 16 of the "Crash Course in Good and Bad Controls" (papers.ssrn.com/sol3/papers.cf...). Here X = class size, Y = math4, Z = read4, and U = student's ability. Conditioning on Z opens the path X -> Z <- U -> Y and it is thus a "bad control."



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What About Multiple Controls?

When considering multiple controls, the status of a single control as "good" or "bad" may change depending on the context of the other variables under consideration.

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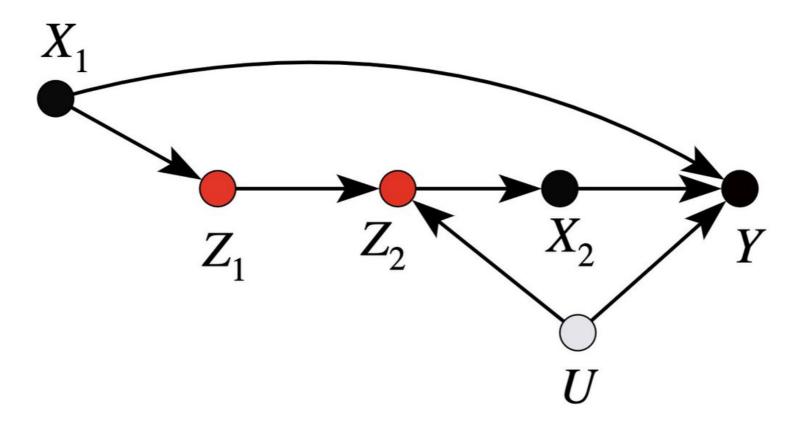
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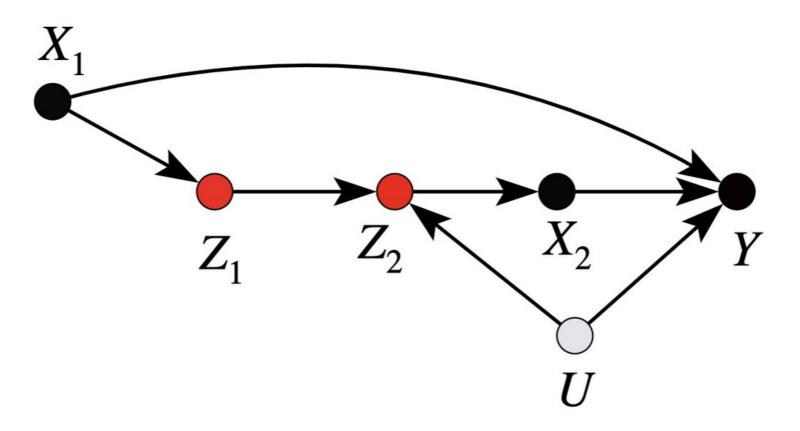
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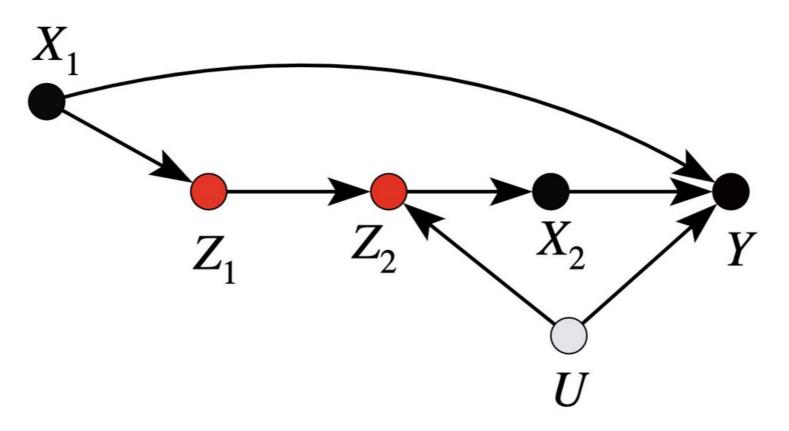
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- You don't need to do it by hand!
- There is open-source software with efficient procedures to identify (optimal) adjustment sets for you (*dagitty, causal fusion, etc*).



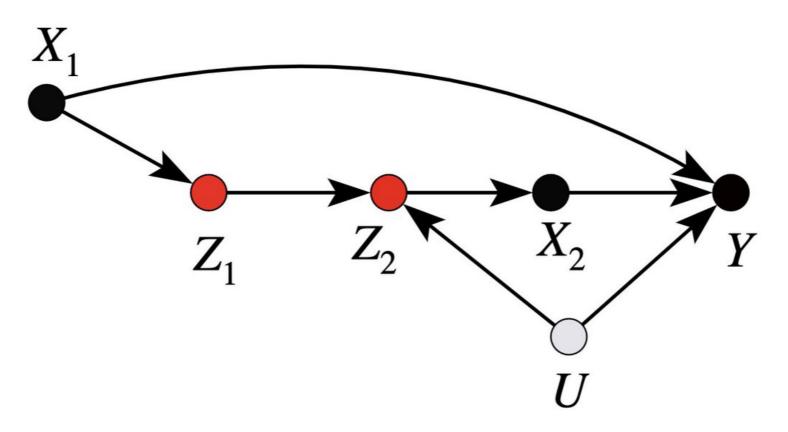


Goal: estimate the *joint effect* of X1 and X2 on Y.



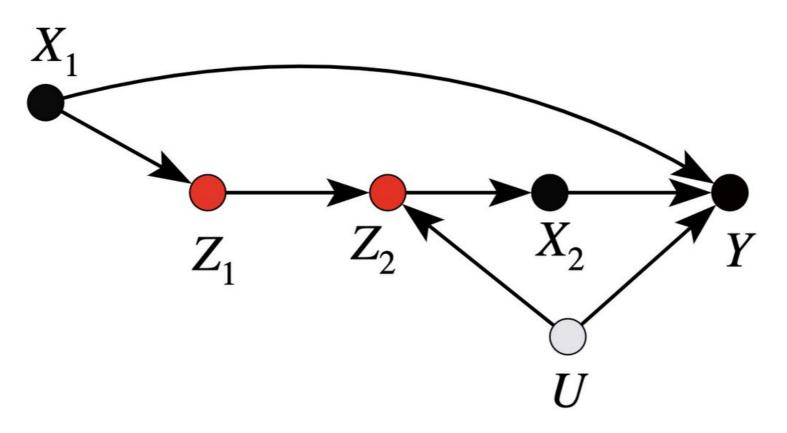
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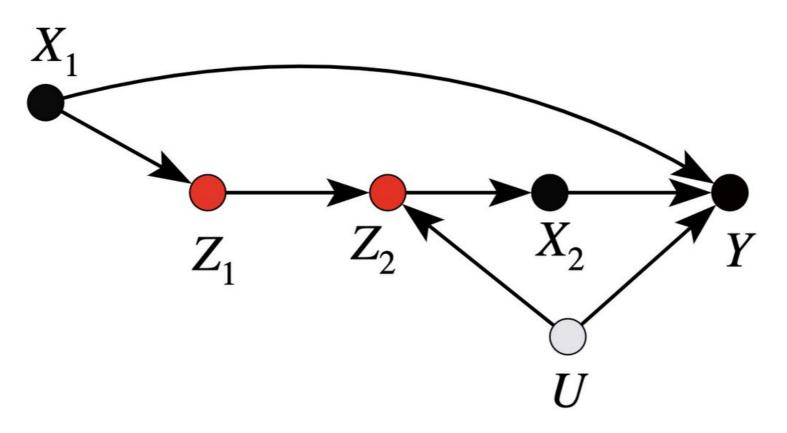


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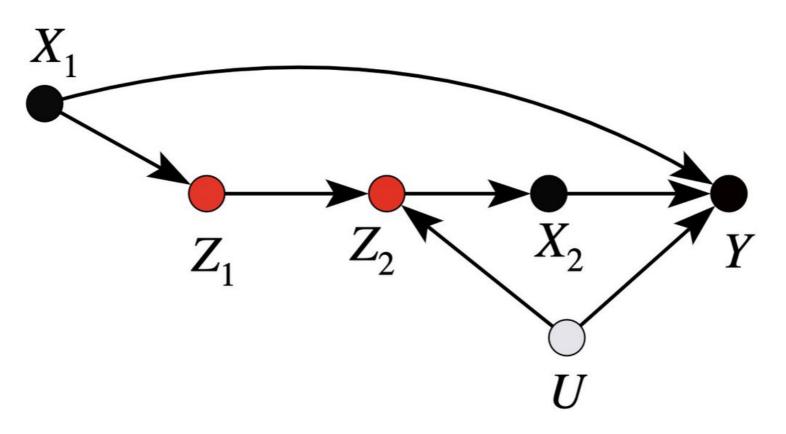
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Answer: include both Z1 and Z2. Note again another example where post-treatment variables are *necessary* for identification.

We have seen through several illustrative examples how simple graphical criteria can be used to decide when a variable should (or should not) be included in a regression equation—and thus whether it can be deemed a "good" or "bad" control.

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Graphical models provide a natural language for <u>articulating</u> such knowledge, as well as efficient tools for examining its <u>logical ramifications</u>.

Thank you!

<u>carloscinelli.com</u>